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# Accounting Price of an Exhaustible Resource: Response and Extensions

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**Abstract** Wei (Environ Resour Econ 60:579–581, 2015) presents a novel derivation of the accounting price for an exhaustible resource in a non-optimal economy subject to an allocation mechanism. We show that Wei (2015) and Hamilton and Ruta (Environ Resour Econ 42:53–64, 2009) are in fact employing different and mutually exclusive allocation mechanisms for the economy, and this explains the differences between the respective accounting prices. Because accounting prices must be defined subject to the allocation mechanism for the economy, the prices derived in the two papers are equally valid within their respective allocation domains. Further analysis shows that if there is declining marginal product of factors, a 'Hartwick investment rule' for the model economy (set investment just equal to depletion, valued at the accounting price) will lead to declining consumption for the Wei (2015) accounting price, and increasing consumption for the Hamilton and Ruta (2009) accounting price. This result is extended to consider the accounting standards recommended in the UN SEEA (System of environmental-economic accounting 2012: central framework. United Nations, European Commission, Food and Agriculture Organization of the United Nations, International Monetary Fund, Organisation for Economic Co-operation and Development, World Bank, 2012), as well as accounting for environmental externalities from resource use.

**Keywords** Exhaustible resource · Accounting price · Resource depletion · Hartwick rule · Environmental externality

**JEL Classification** Q01 · Q32 · Q53 · Q56

The comments of peer reviewers are acknowledged with thanks. The usual caveats apply.

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## 1 Introduction

Hamilton and Ruta (2009) set out a model of a simple extractive economy with the aim of establishing the *accounting price* of an exhaustible resource (this appears in Sect. 5 of their paper). If $S$ is a finite stock of an exhaustible resource and $N$ is its economic value (the present value of the rents generated over the finite lifetime of the resource), then the accounting price of the resource is equal to its marginal social value as measured by $\frac{\partial N}{\partial S}$. As Dasgupta and Mäler (2000) show, accounting prices are the key to measuring sustainability in a non-optimal economy subject to an *allocation mechanism*.[1]

Wei (2015) offers an alternative way to define the accounting price for this extractive economy and suggests that the price derived by Hamilton and Ruta is incorrect. The purpose of this note is to show that Hamilton and Ruta (2009) and Wei (2015) are employing different and mutually exclusive allocation mechanisms, which explains why the accounting prices differ between the two papers. More importantly, this note extends the two papers to establish whether the alternative accounting prices can support a version of the Hartwick rule (Hartwick 1977) in the non-optimal extractive economy. Comparisons with the accounting standard established in the UN system of environmental-economic account (SEEA 2012) are derived, as well as an extension of SEEA (2012) to deal with pollution externalities.

We first clarify the allocation mechanisms in Wei (2015) and Hamilton and Ruta (2009). Section 3 explores the Hartwick rule under the alternative assumptions made in the two papers. Section 4 relates Hamilton's (2016) analysis of the SEEA (2012) to the measurement of sustainability in Hamilton and Ruta (2009), and extends the analysis to include a pollution externality. The final section concludes.

## 2 Alternative Allocation Mechanisms

For an extractive economy with initial stocks of produced assets $K(0)$ and exhaustible resource $S(0)$, an allocation mechanism $\alpha$ defines a mapping

$$\alpha : \{K(0), S(0)\} \rightarrow \{K(t), S(t), R(t), C(t)\}_{t=0}^{\infty}$$

Over this possibly infinite time horizon the paths of produced capital $K$, resource stock $S$, resource extraction $R$ and consumption $C$ are uniquely defined by $\alpha$.[2] The allocation mechanism is *feasible* if $K(t) \geq 0$, $S(t) \geq 0 \; \forall t$.

To make this more concrete, it is useful to take a subset of $\alpha$ that concerns only the evolution of the resource stock and its economic value $N$. The basic accounting rule for the resource is,

$$\dot{S} = -R \tag{1}$$

In Wei (2015) the allocation mechanism $\alpha^W$ given resource stock $S(t)$ consists of:

(W1) Choose an extraction path such that $R(t) = \bar{R}^W$ is constant
(W2) Assume that the unit resource rent is also constant, $n(t) = \bar{n}^W$

[1] Roughly speaking, an allocation mechanism is an algorithm or set of rules that maps initial endowments of assets into a unique future path for the economy. The allocation need not be optimal.

[2] In what follows, all variables are assumed to be functions of time, unless otherwise stated.


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Under these assumptions it follows, as shown in Wei (2015), that the date of exhaustion $T$ of the resource stock is a function of $S(t)$ given by,

$$T(S(t)) = \frac{S(t)}{\overline{R^W}} + t \tag{2}$$

and the value of the resource stock, given constant discount rate $r$ is,

$$N^W = \int_t^{T(S(t))} \bar{n}^W \bar{R}^W \cdot e^{-r(z-t)} dz \tag{3}$$

As Wei establishes, under this allocation mechanism the accounting price is measured as,

$$q \equiv \frac{\partial N^W}{\partial S} = \bar{n}^W \cdot e^{-r(T(S(t))-t)} \tag{4}$$

In Hamilton and Ruta (2009) the allocation mechanism $\alpha^{HR}$ given resource stock $S(t)$ is defined as,

(HR1) Choose a fixed exhaustion time $T$
(HR2) Choose constant $\bar{R}^{HR}$ to satisfy $S(t) = \int_t^T \bar{R}^{HR} dz$
(HR3) Assume a constant unit resource rent $n(t) = \bar{n}^{HR}$

From assumption (HR2) it follows that

$$\bar{R}^{HR} = \frac{S(t)}{T-t} \tag{5}$$

Under these assumptions the value of the resource stock is

$$N^{HR} = \int_t^T \bar{n}^{HR} \bar{R}^{HR} \cdot e^{-r(z-t)} dz = \bar{n}^{HR} \cdot \frac{S(t)}{T-t} \cdot \int_t^T e^{-r(z-t)} dz \tag{6}$$

As Hamilton and Ruta show, the accounting price under this allocation mechanism is

$$p \equiv \frac{\partial N^{HR}}{\partial S} = \frac{\bar{n}^{HR}}{T-t} \cdot \frac{1}{r} \cdot \left(1 - e^{-r(T-t)}\right) \tag{7}$$

As expressions (3) and (6) show, the effect of the alternative allocation methods is to make the resource value $N$ an explicit function of the resource stock $S$.

It is worth exploring the intuition behind these results for alternative accounting prices. Under $\alpha^W$ an increment to the resource stock $\Delta S$ must result in an extension of the exhaustion date to $T(S(t)) + \Delta T$, owing to the fixed quantity of resource extraction at each point in time (W1). The result is that the change in resource value $\Delta N$ is effectively the present value of the last unit of the resource extracted—this is the interpretation of accounting price $q$ as seen in expression (4). Conversely, under $\alpha^{HR}$ the exhaustion date is fixed (HR1). As a result a small increment $\Delta S$ in the resource stock does not affect the accounting price $p$ (expression 7) and the change in the value of the stock is given by $\Delta N = p\Delta S$.

As should be clear, these allocation mechanisms are mutually exclusive. You either choose a fixed quantity of extraction at the outset, which makes the exhaustion time a function of the stock of resource (Wei), or you choose a fixed exhaustion time at the outset, which makes the fixed quantity extracted a function of the stock of resource (Hamilton and Ruta).

As should also be clear, if both allocation mechanisms use the same constant unit rent, say $\bar{n}$, then it is possible to choose extraction $\bar{R} = \bar{R}^W = \bar{R}^{HR}$ such that the exhaustion time is $T = \frac{S(t)}{\bar{R}^W} + t$. In this case the value of the resource stock is the same under either

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approach, $N^W = N^{HR}$, but the two accounting prices $q$ and $p$ are still distinct because they are defined *subject to their corresponding allocation mechanisms*.

Assuming equal values of the resource stock under each approach permits us to analyze the relationship between the two accounting prices. For Wei (2015) we have,

$$q\dot{S} = -qR = \dot{N} \tag{8}$$

$$qS = \dot{N} \cdot (T(S(t)) - t) \tag{9}$$

For Hamilton and Ruta (2009) we have,

$$pS = N \tag{10}$$

$$p\dot{S} + \dot{p}S = \dot{N} \tag{11}$$

Combining expressions (8) and (11) we derive,

$$p = q + \frac{\dot{p}S}{\bar{R}} \tag{12}$$

Expression (8) is derived in Wei (2015), while expression (11) is derived in Hamilton and Ruta (2009). As expressions (8) and (9) suggest, accounting price $q$ only gives economically meaningful values when multiplied by a flow rather than a stock. Expression (11) embodies both the real change in resource wealth defined by Hamilton and Ruta, $p\dot{S} = -pR$, and the corresponding capital gain linked to resource extraction, $\dot{p}S$. Expression (12) is particularly helpful, because it says that the Hamilton and Ruta accounting price $p$ is equal to the Wei accounting price $q$ plus the capital gain per unit of extraction.

It is worth noting that in valuing exhaustible resources, national accountants generally calculate running averages for recent annual quantities of resource extracted and forecast that the most recent average is the constant quantity that will be extracted up to the point of exhaustion of the resource. The time to exhaustion $T - t$ is then calculated as the ratio of economic reserves to the forecast annual quantity extracted. This approach notwithstanding, the implication of the foregoing analysis is that the accountant must assume either that the quantity extracted is fixed or that the terminal date is fixed in determining the accounting price for the resource. The next section shows that this choice has important consequences when applying policies for sustainability in an extractive economy.

## 3 Alternative Implementations of the Hartwick Rule

Hartwick (1977) establishes that a closed extractive economy with fixed technology, constant population and a finite resource that is a necessary input to production can enjoy constant consumption over an infinite horizon if investment in produced capital just equals the value of resource depletion at each point in time.[3] The economy is sustainable under this rule. In the Hartwick model, resource depletion equals the marginal rental value of the resource, and the marginal rental rate is assumed to increase at the rate of interest (the Hotelling Rule).

Here we wish to explore the obvious generalization of the Hartwick rule to the models of Wei (2015) and Hamilton and Ruta (2009). The basic idea is to set investment in produced capital equal to the value of resource depletion derived from the respective allocation mech-

[3] In addition, the elasticity of substitution between produced capital and the exhaustible resource must be equal to 1 in the Hartwick (1977) model, and the elasticity of output with respect to produced capital must be greater than the elasticity for the resource.



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anisms.[4] We generalize the models to an economy with a neoclassical production function such that,

$$F(K,R) = C + \dot{K} + f(R) \tag{13}$$

$$F_K > 0, F_{KK} < 0, F_R < 0, F_{RR} < 0 \tag{14}$$

The production function exhibits declining marginal product with respect to factors, and $f(R)$ is the extraction cost function for the resource. The interest rate $F_K$ is not assumed to be constant over time, which has implications for the results which follow. First, the general expression for the value of the resource stock becomes,

$$N = \bar{n}\bar{R} \cdot \int_t^T e^{-\int_z^t F_K(\tau)d\tau} dx \tag{15}$$

Under Wei's allocation mechanism the accounting price is therefore,

$$q = \bar{n}e^{-\int_t^{T(S(t))} F_K(\tau)d\tau}$$

Under Hamilton and Ruta's allocation mechanism the accounting price is,

$$p = \frac{\bar{n}}{T-t} \cdot \frac{1}{F_K(t)} \cdot \left(1 - e^{-\int_t^T F_K(\tau)d\tau}\right)$$

Finally, from expression (15) we derive the instantaneous change in the value of the resource stock as a result of resource extraction,

$$\dot{N} = -\bar{n}\bar{R} \cdot e^{-\int_t^T F_K(\tau)d\tau} \tag{16}$$

We can now extend the analysis in the preceding section by introducing a "Hartwick investment rule" into the allocation mechanisms of Wei (2015) and Hamilton and Ruta (2009). Expression (13) indicates that output can be consumed, invested or spent on resource extraction. Since allocation rules W1 and HR2 specify the path for resource extraction, they determine extraction costs $f(R)$. The introduction of an investment rule into the respective allocation mechanisms therefore determines the future path of consumption, yielding a unique future path for the economy as a whole.

For the model of Wei the investment rule is

(W3) $\dot{K} = q\bar{R} = -\dot{N}$

Since $\bar{R}$ is assumed to be constant (W1), it follows from expression (13) that,

$$\ddot{K} = F_k\dot{K} - \dot{C} = -\frac{d}{dt}\left(\dot{N}\right) = -\frac{d}{dt}\left(F_kN - \bar{n}\bar{R}\right) = -\dot{F}_kN - F_k\dot{N}$$

And therefore, substituting (W3),

$$\dot{C} = \dot{F}_kN \tag{17}$$

For the model of Hamilton and Ruta the investment rule is

(HR4) $\dot{K} = p\bar{R} = \frac{N}{S} \cdot \bar{R}$

From (HR4) we have

$$\ddot{K} = \frac{\dot{N}S + N\bar{R}}{S^s} \cdot \bar{R} = \frac{F_KN - \bar{n}\bar{R}S + N\bar{R}}{S^2} \cdot \bar{R}$$

[4] The analysis in this section is a special case (assuming constant extraction and constant unit rent) of the more general results in Hamilton (2016). We make comparisons with Hamilton (2016) in Sect. 4 below.

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Given $\bar{R}$ constant (HR2), it follows that,

$$\dot{C} = F_K \dot{K} - \ddot{K} = F_K \frac{N}{S} \cdot \bar{R} - \frac{F_K N - \bar{n}\bar{R}S + N\bar{R}}{S^2} \cdot \bar{R}$$

Collecting terms this reduces to,

$$\dot{C} = (\bar{n}S - N) \cdot \frac{\bar{R}^2}{S^2} \tag{18}$$

The result for the Wei allocation mechanism, expression (17), is negative under the assumption of declining marginal product of factors.[5] Investing $q\bar{R}$ at each point in time results in declining consumption. Conversely, since $\bar{n}S$ is the undiscounted value of resource flows over the life of the resource, and $N$ is the discounted present value of these resource flows, expression (18) establishes that the Hamilton and Ruta allocation mechanism yields growing consumption at each point in time.

## 4 Extensions: The UN SEEA (2012) and Externalities from Resource Use

Having established how the Wei (2015) and Hamilton and Ruta (2009) models perform under a Hartwick investment rule, these results can be made more general by considering how the Hamilton and Ruta model relates to the System of Environmental-Economic Accounting (SEEA 2012). A further extension analyzes how to account for an environmental externality from resource use, combining the assumptions of fixed exhaustion time $T$ and constant resource extraction $\bar{R}$ with SEEA (2012) accounting conventions.

An important step in standardizing environmental and resource accounting practice was the adoption of the SEEA (2012) as a UN statistical standard. Hamilton (2016) analyzes the application of SEEA (2012) to the problem of measuring the sustainability of development, using the generalized Hartwick rule of Hamilton and Hartwick (2005).[6]

Expression (10) above implies that the unit value of depletion of an exhaustible resource in Hamilton and Ruta (2009) is given by $p \equiv \frac{N}{S}$. This corresponds exactly to the standard set out in SEEA (2012), and so it is useful to compare the measurement of sustainability presented in Sect. 3 to that derived in Hamilton (2016) for the SEEA. There are two differences between the models of Hamilton and Ruta (2009) and Hamilton (2016). First, Hamilton (2016) assumes that resource extraction declines at a constant rate, $\frac{\dot{R}}{R} = -\phi$; this compares with the constant level of extraction $\bar{R}$ associated with a fixed exhaustion time $T$ in Hamilton and Ruta. Second, Hamilton (2016) assumes that marginal resource extraction costs are constant, which implies that unit rents will vary with the resource price $F_R$; Hamilton and Ruta assume that unit rents $\bar{n}$ are constant.

Hamilton (2016) shows that, given the assumptions about declining extraction and constant marginal extraction costs, setting genuine saving $G \equiv \dot{K} - pR$ equal to 0 (the standard Hartwick rule) implies that consumption is instantaneously constant. If, instead, there are increasing marginal resource extraction costs then the standard Hartwick rule implies that $\dot{C}$ is proportional to the (positive) inframarginal rents on extraction; the standard Hartwick rule implies increasing consumption in this instance.

5 To be precise, (W3) implies that $\dot{K} > 0$ since $\dot{N} < 0$ (expression 16), and so $\dot{F}_K = F_{KK}\dot{K} < 0$.

6 Hamilton and Hartwick (2005) derive a generalized Hartwick rule, showing that $\dot{C} = F_K G - \dot{G}$ for genuine saving $G$. The standard Hartwick rule is a special case for $G = \dot{G} = 0$; a path where genuine saving is identically 0 at each point in time will exhibit constant consumption.


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Expression (18) above shows that the standard Hartwick rule in the Hamilton and Ruta (2009) model also implies increasing consumption. An important advantage evident in expression (18), however, is that it is simple to measure the amount by which consumption increases under the standard Hartwick rule. In contrast, to measure this increase in the Hamilton (2016) model with increasing marginal extraction costs requires specification of the extraction cost function, knowledge that may not be forthcoming in many circumstances.

### 4.1 An Environmental Externality from Resource Use

If we assume that healthfulness $H$ is a stock that contributes to wellbeing, then utility can be measured as $U = U(C, H)$. In what follows we examine the question of measuring sustainability using SEEA (2012) conventions when extraction of the resource leads to health damage measured as $d\left(\bar{R}\right)$; formally, $d\left(\bar{R}\right)$ is represented as a deduction from the stock of health $H$, while extraction $\bar{R}$ is held constant for an assumed exhaustion date $T$.

The optimal growth problem is to maximize

$$V = \int_t^T U(C, H) \cdot e^{-\rho(s-t)} ds \tag{19}$$

subject to accounting identity (13) and the following:

$$\dot{H} = -d\left(\bar{R}\right). \tag{20}$$

$$\dot{S} = -\bar{R} \tag{21}$$

If we think of local air pollution as an example of an externality associated with use of the resource in production, then $d\left(\bar{R}\right)$ encompasses the whole sequence from resource use to pollutant emission, dispersion, and human exposure, finally resulting in damage to health. In what follows we make the simplifying assumption that the marginal extraction cost for the resource $f'$ is constant, as is the marginal health damage from resource use $d'$.[7] We therefore relax the assumption of constant unit resource rents in Wei (2015) and Hamilton and Ruta (2009).

The basic growth theory leading to the expression for genuine saving in an optimal extractive economy with a pollution externality is derived in the "Annex: Optimal Resource Extraction with a Health Externality" section. Taking the results for the optimal economy as a model, we derive the generalized Hartwick rule for the non-optimal economy under the assumptions of constant resource extraction and SEEA accounting conventions for measuring resource depletion.

A key parameter derived in the "Annex: Optimal Resource Extraction with a Health Externality" section is $z$, the shadow price of a unit of health damage, which equals the present value of the instantaneous willingness to pay for a unit of healthfulness $U_H/U_C$, as seen in "Annex: Optimal Resource Extraction with a Health Externality" section expression (27). Because of the externality, the shadow price of the resource has to include the value of marginal damage to health from resource use, as seen in the "Annex: Optimal Resource Extraction with a Health Externality" section expression (26). Because marginal extraction costs and marginal health damages are constant, the value of the resource stock $N$ is equal to the present value of net resource rents,

$$N = \int_t^T \left(F_R - f' - zd'\right) \bar{R} \cdot e^{\int_t^z f_k(\tau)d\tau} dz$$

[7] This simplification is not necessary, but it streamlines the derivation of the main results.

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This corresponds to expression (15) in Sect. 3. Genuine saving in this economy is given by,

$$G \equiv \dot{K} - pR - zd \tag{22}$$

That is, net saving equals investment in produced capital, minus depletion of the resource stock, minus the value of damage to the stock of health from the pollution externality. Here $p \equiv N/S$, per SEEA (2012) conventions. The generalized Hartwick rule is derived as follows:

$$\begin{aligned}
\dot{G} &= \ddot{K} - \dot{z}d - \dot{p}\bar{R} \\
&= F_K\dot{K} - \dot{C} - \left(F_K z - \frac{U_H}{U_C}\right)d - \left(F_k p - \left(F_R - f' - zd'\right)\right)\frac{R}{S} + N\frac{R}{S^2} \\
F_K G &= F_K\dot{K} - FkpR - F_k zd
\end{aligned}$$

Collecting terms as in Sect. 3 yields the generalized Hartwick rule:

$$\dot{C} - \frac{U_H}{U_C}d = F_k G - \dot{G} + \left(\left(F_R - f' - Zd'\right)S - N\right)\frac{R^2}{S^2} \tag{23}$$

In this expression $\dot{C} - \frac{U_H}{U_C}d$ is the dollar-valued instantaneous change in wellbeing (taking account of the health damages from resource use). The term $\left(\left(F_R - f' - zd'\right)S - N\right)$ is the difference between the value of the resource stock in the optimal economy[8] and $N$, which is the value of the resource stock in the non-optimal economy being modeled. The final term in expression (23) is therefore positive. Under the standard Hartwick rule $\left(G = \dot{G} = 0\right)$ wellbeing is therefore increasing, closely paralleling Sect. 3 with the exception that the health externality has to be included. If genuine saving is non-negative and growing at a rate less than the interest rate $F_K$ at each point in time, then wellbeing is everywhere increasing, which implies that social welfare $V$ is also everywhere increasing.

## 5 Conclusions

Wei's 2015 contribution to the literature is to show that there is an alternative allocation mechanism that can be applied to the model economy of Hamilton and Ruta (2009), and that this mechanism leads to a novel accounting price for the resource. The foregoing analysis shows that the allocation mechanisms employed in each paper are distinct and the accounting prices derived are equally valid within the domain defined by their respective allocation mechanisms. In this response we have derived the relationship between the accounting prices in the two papers.

Neither Wei (2015) nor Hamilton and Ruta (2009) go on to establish the behavior of the economy under a "Hartwick investment rule" derived from their respective accounting prices for the resource. This response to Wei fills the gap by showing that, under standard assumptions regarding declining marginal product of factors, the Wei accounting price results in declining consumption at each point in time, while the Hamilton and Ruta accounting price leads to increasing consumption. If the policy goal for the economy is to achieve constant consumption, then the Wei approach is clearly under-investing while the Hamilton and Ruta approach is over-investing. If the policy goal is non-declining consumption at each point in time, then the Hamilton and Ruta approach is a sufficient condition to reach the goal.

[8] Recall that the Hotelling rule, expression (26), applies in the optimal economy. As a result the growth in unit rents is completely offset by the discount rate.



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Looking forward, it is clear that much more rigor is required in the application of Dasgupta and Mäler (2000) concept of accounting prices for assets. In particular, it needs to be made clear for models such as the above that $\frac{\partial N}{\partial S} \equiv \frac{\partial}{\partial S} N(S; \alpha)$. That is, accounting prices can only be measured with respect to the assumed allocation mechanism $\alpha$. And $\alpha$ needs to be fully specified.

We also show that the Hamilton and Ruta (2009) model of extraction implicitly employs the valuation of resource depletion adopted in SEEA (2012). As a result, there are strong parallels between the standard Hartwick rule, derived using the Hamilton and Ruta (2009) assumptions, and the Hamilton (2016) analysis of the standard Hartwick rule using SEEA (2012) accounting conventions.

If resource use results in health damages (for example from pollution emissions), then we show that under SEEA (2012) accounting conventions the generalized Hartwick rule has to account both for instantaneous health damages and the reduction in resource rents resulting from the persistent loss of healthfulness associated with resource use. In this model the standard Hartwick rule leads to increasing wellbeing.

## Annex: Optimal Resource Extraction with a Health Externality

For health stock $H$, health damage function $d(R)$, utility $U(C, H)$ and accumulation equations (13), (20) and (21), the objective is to maximize,

$$V = \int_t^T U(C, H) \cdot e^{-\rho(s-t)} ds \tag{24}$$

for constant pure rate of time preference $\rho$. The Hamiltonian function is given by,

$$\mathcal{H} = U + \gamma_1 \dot{K} + \gamma_2 \dot{H} + \gamma_3 \dot{S}$$

where the $\gamma_i$ are the corresponding shadow prices. From the first order condition on consumption ($\frac{\partial U}{\partial C} = 0$) it follows that $\gamma_1 = U_C$, while the dynamic first order condition $\left(\dot{\gamma}_1 = \rho\gamma_1 - \frac{\partial H}{\partial K}\right)$ on $\dot{U}_C$ yields the standard Ramsey equation,

$$F_K = \rho - \left(\frac{\dot{U}_C}{U_C}\right) \tag{25}$$

Defining $\gamma_2 \equiv U_C z$, where $z$ is the value of a unit of the health stock $H$, the first order condition on extraction yields,

$$\gamma_3 = U_C \left(F_R - f' - zd'\right)$$

The dynamic first order condition for $\dot{\gamma}_3$ therefore gives the Hotelling rule for this economy,

$$\frac{\frac{d}{dt}\left(F_R - f' - zd'\right)}{F_R - f' - zd'} = F_K \tag{26}$$

Marginal rents on extraction therefore deduct the marginal damage to health $zd'$. Next, the dyamic first order condition on $\dot{\gamma}_2$ gives,

$$\frac{d}{dt}(U_C z) = \rho U_C z - U_H$$

and substituting (26) yields,

$$\dot{z} = F_K z - \frac{U_H}{U_C}$$



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This differential equation has solution,

$$z = \int_t^{\infty} \frac{U_H}{U_C}(s) \cdot e^{-\int_t^s F_K(\tau)d\tau} ds \tag{27}$$

Genuine saving $G$ is therefore derived from the Hamiltonian function as,

$$G = \dot{K} - zd(R) - \left(F_R - f' - zd'\right) R \tag{28}$$

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